Barone-Adesi Whaley (1987) Pricing Formulas Review

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1 European Option Formulas

Define

$$d_1 = \frac{\ln(S/X) + (b + 0.5\sigma^2)T}{\sigma\sqrt{T}}, \qquad d_2 = d_1 - \sigma\sqrt{T}.$$

European call and put (eqs. (5)–(6)):

$$c(S,T) = Se^{(b-r)T}N(d_1) - Xe^{-rT}N(d_2),$$

$$p(S,T) = Xe^{-rT}N(-d_2) - Se^{(b-r)T}N(-d_1).$$

2 Auxiliary Quantities

$$K(T) \equiv 1 - e^{-rT}, \qquad M \equiv \frac{2r}{\sigma^2}, \qquad N \equiv \frac{2b}{\sigma^2}.$$

3 Power Exponents

The quadratic (paper eq. (13)):

$$q^2 + (N-1)q - \frac{M}{K(T)} = 0$$

has roots

$$q_1 = \frac{-(N-1) - \sqrt{(N-1)^2 + 4M/K(T)}}{2}, \qquad q_2 = \frac{-(N-1) + \sqrt{(N-1)^2 + 4M/K(T)}}{2}.$$

For calls use $q_2 > 0$. For puts use $q_1 < 0$.

4 American Call Approximation

If $b \geq r$,

$$C(S,T) = c(S,T).$$

If $b < r$, the Barone–Adesi & Whaley approximation is

$$C(S,T) = \{\, c\ (S,T) + A_2\,(SS^*)^{q_2}\ , S < S^*, S - X, S \geq S^*,$$

where S^* solves (eq. (19)):

$$S^* - X = c(S^*,T) + \frac{1 - e^{(b-r)T} N\big(d_1(S^*)\big)}{q_2}\, S^*.$$

Given S^*,

$$A_2 = \frac{S^*}{q_2}\left[1 - e^{(b-r)T} N(d_1(S^*))\right].$$

5 American Put Approximation

The put price is

$$P(S,T) = \{\, p\ (S,T) + A_1\,(SS^{**})^{q_1}\ , S > S^{**}, X - S, S \leq S^{**},$$

where S^{**} solves (eq. (24)):

$$X - S^{**} = p(S^{**},T) - \left[1 - e^{(b-r)T} N(-d_1(S^{**}))\right] \frac{S^{**}}{q_1}.$$

Given S^{**},

$$A_1 = -\frac{S^{**}}{q_1}\left[1 - e^{(b-r)T} N(-d_1(S^{**}))\right].$$

6 Practical Computation Recipe

1. Compute $c(S,T)$, $p(S,T)$ from the European formulas.
2. Compute $K(T) = 1 - e^{-rT}$, $M = 2r/\sigma^2$, $N = 2b/\sigma^2$ and roots q_1, q_2.
3. For the call:
   - If $b \geq r$, set $C = c$.
   - If $b < r$, solve for S^* from

     $$S^* - X = c(S^*,T) + \frac{1 - e^{(b-r)T} N(d_1(S^*))}{q_2} S^*,$$

     then compute $A_2 = \frac{S^*}{q_2}\big[1 - e^{(b-r)T} N(d_1(S^*))\big]$, and evaluate the piecewise call formula.

4. For the put:

   - Solve for S^{**} from

     $$X - S^{**} = p(S^{**}, T) - \left[1 - e^{(b-r)T} N(-d_1(S^{**}))\right] \frac{S^{**}}{q_1},$$

     then compute

     $$A_1 = -\frac{S^{**}}{q_1} \left[1 - e^{(b-r)T} N(-d_1(S^{**}))\right],$$

     and evaluate the piecewise put formula.